

## Mathematics and Critical Thinking, Part II

In the previous article I began telling the story of an unusual high school geometry course run at the Ohio State University in 1930s. The course has been designed and taught by Prof. Harold F. Fawcett who later published an account in *The Nature of Proof* (NCTM, 13th Yearbook, Reprint 1995). To quote from the book, "There has probably never been a time in the history of American education when the development of critical and reflective thought was not recognized as desirable outcome of the secondary school. In Fawcett's view, geometry was the most suitable course in the secondary school to teach critical and reflective thinking. He provides a respectable selection of quotes to support his view and to explain the source of his dissatisfaction with the traditional courses. Traditionally, ... the major emphasis is placed on a body of theorems to be learned rather than on the method by which these theorems are established. As the result, ... there is little evidence to show that pupils who have studied demonstrative geometry are less gullible, more logical and more critical in their thinking than those who did not follow such a course. The worthy outcome for students taking a geometry course is not only proving and learning a set of theorems, but acquiring of mental habits that save them from floundering in the conduct of life. Not only students should learn to prove a number of theorems but also grasp the nature of proof, so that their analytic ability could be transferred to non-geometric situations. And how is this achieved? Fawcett cites the prevalent view point "No transfer will occur unless the material is learned in connection with the field to which the transfer is desired. ... Transfer is not automatic. We reap no more than we sow ... Fawcett concludes that transfer is secured only by training for transfer, which explains the unconventional opening of his course (see Part I). Next he deals with methods and procedures suitable for such a study. His treatment is so much pertinent to the modern day discussions (minding children's own logic and individual ways, group discussions, open ended approach, discovery and investigation) that Fawcett's experiment and the book deserve to be better known among math educators. The point of the opening discussion was to establish the need in agreed-upon definitions, which seemed foreign to the thinking of the pupils. For example, at the outset all students agreed that "Abraham Lincoln spent very little time in school" and no one raised the point that the truth of this statement depends on how "school" is defined. So, starting with the first meeting, students were led to recognize the importance of definitions and, later, the need in undefined terms. They were taught to recognize the presence of implicit assumptions even in the most elementary activities of life. Flener interviewed Warren Mathews, a course graduate. Mathews' comments were, "I remember all our work with definitions. When I was a vice president at Hughes, and now in my work with my church, I realize how important definitions are. It is amazing that when we can agree on our definitions most of the conflict ends. To which Flener remarked "In the field of education we probably argue at cross purposes more because we don't have the same definitions in mind. How true! And how sad! Except of course math educators have no particular reason to feel singled out in this respect. "In the field of education" should be considered as a generic designation. But let's try to apply the course basics to the course itself. Is it correct to designate Fawcett's experiment the geometry course? What makes a school year long interaction of a group of students with one or more teachers a geometry course? Can you think of a suitable definition? How does it jibe with Fawcett's remark [p. 102 of his book]? While the control of geometric subject matter was not one of the major purposes to be accomplished by the pupils in class A, nevertheless it seemed desirable to compare their achievement in this respect with that of pupils who had followed the usual course in geometry. I think that the description "Critical Thinking Course with Applications to Geometry" serves well the purpose, the proceedings, and the results of Fawcett's course. The skill transfer occurred in the direction opposite to the declared goal! What Fawcett's experiment demonstrates very convincingly is that development of critical thinking skills helps students master mathematics even when they feel no particular liking for the subject. At the end of the course, Fawcett interviewed students' parents. In their parents' view, the course helped 16 students improve their ability to think critically, but only 3 out of a little more than 20 students have learned to like mathematics. And so what? In an 1997 article *Is Mathematics Necessary?*, Underwood Dudley argues that the answer to the question in the title of his paper is a sound No. He ends the article with a pun, "Is mathematics necessary? No. But it is sufficient. This may or may not be so. But, in any event, some things seem to be more sufficient than others. (A discussion on what mathematics may be sufficient for, could have fit right in with Fawcett's geometry course.) We just saw how the critical thinking skills helped study mathematics. Flener too ties the success of the course to the fact that students who took the course were the University School veterans of three years and were used to open ended investigations. Following is a more complete quote from Dudley's paper: "Can you recall why you fell in love with mathematics? It was not, I think, because of its usefulness in controlling inventories. Was it not because of the delight, the feeling of power and satisfaction it gave; the theorems that inspired awe, or jubilation, or amazement; the wonder and glory of what I think is the human race's supreme intellectual achievement? Mathematics is more important than jobs. It transcends them, it does not need them. Is mathematics necessary? No. But it is sufficient. No doubt mathematician Fawcett knew about and could appreciate the glory and the beauty of mathematics. He was an outstanding teacher and could, if he wanted to, do a better job passing on to his students this sense of beauty and amazement shared by all mathematicians. He apparently chose not to. His goal was to teach the students, via interaction with mathematics, critical and reflective thought. But the goals of education are many: acquisition of useful skills, absorption of the local and global cultures, development of the innate potential. Course offerings could and should match a variety of goals. It stands to reason that the manner in which a math course is planned and conducted should aim at a particular objective. There is no single right way to teach and study mathematics. The definitions are important. To resolve the cross purpose discussions, it's no less important to accept a possibility that an approach may be as right, or as good, as another one - perhaps for a different end. (This article is an adaptation of an 2001 MAA Online column available at <http://www.cut-the-knot.org/ctk/NecessaryAndSufficient.shtml>.)

## About the Author

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